

**PROBLEMA 1.**

1. Calcular el valor de la integral  $\int_{-\pi}^{2\pi} |x| \operatorname{sen} x \, dx$

La función que hay que integrar:

$$|x| \operatorname{sen} x = \begin{cases} -x \operatorname{sen} x & \text{si } x < 0 \\ x \operatorname{sen} x & \text{si } x \geq 0 \end{cases}$$

Entonces:

$$\int_{-\pi}^{2\pi} |x| \operatorname{sen} x \, dx = \int_{-\pi}^0 -x \operatorname{sen} x \, dx + \int_0^{2\pi} x \operatorname{sen} x \, dx$$

Luego:

$$u = x \rightarrow du = dx$$

$$dv = \operatorname{sen} x \, dx \rightarrow v = -\cos x$$

$$\int x \operatorname{sen} x \, dx = -x \cos x - \int -\cos x \, dx = -x \cos x + \operatorname{sen} x$$

$$\int_0^{2\pi} x \operatorname{sen} x \, dx = [-x \cos x + \operatorname{sen} x]_0^{2\pi} = -2\pi$$

$$\int_{-\pi}^0 -x \operatorname{sen} x \, dx = [x \cos x - \operatorname{sen} x]_{-\pi}^0 = -\pi$$

Luego:

$$\int_{-\pi}^{2\pi} |x| \operatorname{sen} x \, dx = -\pi - 2\pi = -3\pi$$

