CENTRO DE ESTUDIOS MIRASIERRA www.selectividad.net/cem

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INTEGRALES

1.* $\int \frac{k\,x^n}{p}\,dx = \frac{k}{p}\int x^n\,dx = \frac{k\,x^{(n+1)}}{p\,(n+1)} + C \Rightarrow n \neq -1$

2.* $\int f(x)+g(x)-h(x)\,dx = \int f(x)\,dx + \int g(x)\,dx - \int h(x)\,dx$

3.* $\int \frac{x^n}{\sqrt[p]{x^k}}\,dx = \int x^n x^{-k/p}\,dx = \int x^{n-k/p}dx = \int x^L\,dx = \frac{x^{L+1}}{L+1} + C$

4.* $\int f'(x)\,[f(x)]^n\,dx = \frac{[f(x)]^{n+1}}{n+1} + C$

$\left\{ \begin{array}{l} \frac{f'(x)}{[f(x)]^n} = f'(x)[f(x)]^{-n} \\ \frac{f'(x)}{\sqrt[n]{f(x)^p}} = f'(x)[f(x)]^{-p/n} \end{array} \right.$

5.* $\int f'(x)\,k^{f(x)}\,L\,k\,dx = K^{f(x)} + C \Rightarrow Le = 1$ - - - - - - - - - - - - - - - -

Si **Sobran "x" en f '(x) ,** pasamos a resolverla **por partes**

6.* $\int \frac{f'(x)}{f(x)}\,dx = L|f(x)| + C$

7.* $\int \frac{f(x) \pm g(x)}{p(x)}\,dx = \int \frac{f(x)}{p(x)}\,dx \pm \int \frac{g(x)}{p(x)}\,dx$

8.* $\int \frac{f(x)}{g(x)}\,dx \Rightarrow$ si el exponente de f(x) ≥ g(x) ⇒

$$\begin{array}{l|l} \mathbf{f(x)} & \mathbf{g(x)} \\ \hline \mathbf{Rt} & \mathbf{Co} \end{array}$$

$\int \frac{f(x)}{g(x)}\,dx = \int Co + \frac{Rt}{g(x)}\,dx$

Si **no hay soluciones** en los reales **g(x) = 0,** pasamos a **ver si es arc.tang**

9.* $\int \frac{f(x)}{g(x)}\,dx \Rightarrow$ Resolvemos $g(x) = 0 \Rightarrow x_1 = a\ ;\ x_2 = -b \Rightarrow \frac{f(x)}{g(x)} = \frac{A}{x-a} + \frac{B}{x+b}$

$\int \frac{f(x)}{g(x)}\,dx = \int \frac{A}{x-a}\,dx + \int \frac{B}{x+b}\,dx$

Si $g(x) = (x-a)^2(x+b) \Rightarrow \frac{f(x)}{g(x)} = \frac{A}{(x-a)^2} + \frac{B}{x-a} + \frac{C}{x+b}$

Trigonométricas :

$\int f'(x)\cos f(x)\,dx = \operatorname{sen} f(x) + C$

$\int f'(x)\operatorname{sen} f(x)\,dx = -\cos f(x) + C$

$\int \frac{f'(x)}{\cos^2 f(x)}\,dx = \operatorname{tang} f(x) + C$

$\int \frac{f'(x)}{\operatorname{sen}^2 f(x)}\,dx = -\operatorname{cotang} f(x) + C$

$\int \frac{f'(x)}{\sqrt{1-[f(x)]^2}}\,dx = \operatorname{arc.sen} f(x) + C$

$\int \frac{-f'(x)}{\sqrt{1-[f(x)]^2}}\,dx = \operatorname{arc.cos} f(x) + C$

$\int \frac{f'(x)}{1+[f(x)]^2}\,dx = \operatorname{arc.tag} f(x) + C$

$\int \frac{-f'(x)}{1+[f(x)]^2}\,dx = \operatorname{arc.cotag} f(x) + C$

Si **sobran "x" en f´(x)** , pasamos a resolverla **por partes**

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* $\operatorname{sen}^n f(x)$
* $\cos^n f(x)$

- **n ⇒ número impar**: **trabajar con : $\cos^2 f(x) + \operatorname{sen}^2 f(x) = 1$**
  $\operatorname{sen}^2 f(x) = 1 - \cos^2 f(x) \Leftrightarrow \cos^2 f(x) = 1 - \operatorname{sen}^2 f(x)$
- **n ⇒ número par**: **trabajar con : $\cos 2.f(x) = \cos^2 f(x) - \operatorname{sen}^2 f(x)$**
  $\operatorname{sen}^2 f(x) = \frac{1}{2}\left[1 - \cos 2 f(x)\right] \Leftrightarrow \cos^2 f(x) = \frac{1}{2}\left[1 + \cos 2 f(x)\right]$

Por partes:

u = lo que sobre, siempre que no tengamos un logaritmo o un arco

$\int f(x) . g(x)\, dx \qquad u = f(x) \Rightarrow du = f'(x)\, dx$

$dv = g(x)\, dx \Rightarrow v = \int g(x)\, dx$ ⇒ Resolvemos la integral para sacar "v"

$\int \mathbf{f(x) . g(x)\, dx = u . v - \int v . du}$

Se realiza esta operación hasta que la última integral sea inmediata

$\int f(x). g(x)\, dx = \mathbf{I} \Rightarrow \mathbf{I} = [\ u.v - (\quad)\ ] - \frac{2}{3} \int f(x) . g(x)\, dx$

$\mathbf{I} = [\ u.v - (\quad)\ ] - \frac{2}{3}\mathbf{I} \Rightarrow I + \frac{2}{3}\mathbf{I} = [\ u.v - (\quad)\ ]$

$\frac{5}{3}\mathbf{I} = [\ u.v\ (\quad)\ ] \Rightarrow \mathbf{I} = \frac{3}{5}[\ u.v\ (\quad)\ ] + C$

Si al realizar sucesivas partes aparece la integral del principio la llamaremos "**I**" y operamos

Recuerda:

$\operatorname{sen}^2 f(x) + \cos^2 f(x) = 1$ $\qquad$ $\cos^2 f(x) - \operatorname{sen}^2 f(x) = \cos 2 f(x)$

$1 + \operatorname{tang}^2 f(x) = \sec^2 f(x)$ $\qquad$ $2 \operatorname{sen} f(x) \cos f(x) = \operatorname{sen} 2 f(x)$

$1 + \operatorname{cotg}^2 f(x) = \operatorname{cosc}^2 f(x)$ $\qquad$ $\operatorname{sen} f(x) \cos g(x) = \frac{1}{2}\left[\operatorname{sen}[f(x) - g(x)] + \operatorname{sen}[f(x) + g(x)]\right]$

$\operatorname{sen}^2 f(x) = \frac{1}{2}\left[1 - \cos 2 f(x)\right]$ $\qquad$ $\operatorname{sen} f(x) \operatorname{sen} g(x) = -\frac{1}{2}\left[\cos[f(x) - g(x)] - \cos[f(x) + g(x)]\right]$

$\cos^2 f(x) = \frac{1}{2}\left[1 + \cos 2 f(x)\right]$ $\qquad$ $\cos f(x) \cos g(x) = \frac{1}{2}\left[\cos[f(x) - g(x)] + \cos[f(x) + g(x)]\right]$

$\operatorname{tang} f(x) = \frac{\operatorname{sen} f(x)}{\cos f(x)}$ $\qquad$ $1 - \cos f(x) = 2 \operatorname{sen}^2 \frac{f(x)}{2}$

$\cos f(x) = \operatorname{sen}\left[\frac{\pi}{2} - f(x)\right]$ $\qquad$ $1 + \cos f(x) = 2 \cos^2 \frac{f(x)}{2}$